Computation of Turbulent Flows on Rotating Bodies and Ducts

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Abstract

A k-ϵ model for three-dimensional boundary layers modified to include anisotropy and one modified for rotating flows are described. Based on these models, the predictions of a space-marching code show good agreement with the flow data for a rotating cylinder and a rotating channel.

Nomenclature

C_1, C_2	= constants in the algebraic stress model ($C_1 = 1.5$, $C_2 = 0.6$)
D	= semiwidth of the two-dimensional channel
k	= turbulent kinetic energy
P_{ij}	= production of $\overline{u_i u_j}$
P	= production of k
R_0	= rotational parameter $= 2\Omega D/U_{1\text{mean}}$
T	= parameter in the anisotropic eddy viscosity model
U_1, U_2, U_3	= velocity components [U_1 is the principal velocity component, U_2 normal (x_2) to the wall, U_3 in the tranverse (x_3) direction]
U_{1e}	= U_1 velocity at the edge of the boundary layer
u_1, u_2, u_3	= fluctuating components of velocity in x_1, x_2, x_3 directions
x_1, x_2, x_3	= coordinates: x_1 is the main flow direction, x_2 the normal to the wall, and x_3 the cross-flow direction
ϵ	= rate of turbulent kinetic energy dissipation
ρ	= density
Ω, Ω_p	= angular velocity

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Modifications for Anisotropy

In most three-dimensional boundary layers, $\partial U_1/\partial X_2$ and $\partial U_3/\partial x_2$ are the only important mean velocity gradients. Hence, the Reynolds shear stresses can be expressed in the following manner:

$$-\rho\overline{u_1 u_2} = C\mu_{12}\rho\frac{k^2}{\epsilon}\frac{\partial U_1}{\partial x_2} \tag{1}$$

$$-\rho\overline{u_3 u_2} = C\mu_{32}\rho\frac{k^2}{\epsilon}\frac{\partial U_3}{\partial x_2} \tag{2}$$

When $C_{\mu 12} \neq C_{\mu 32}$, the model is anisotropic. In many studies, the isotropic assumption is used with $C_{\mu 12} = C_{\mu 32} = 0.90$.

Starting from Rotta's T model[1] the following expressions are derived for $C_{\mu 12}$ and $C_{\mu 32}$ (see Ref. 2 for details)

$$C_{\mu 12} = 0.09\left[\frac{U_1^2 + TU_3^2}{U_1^2 + U_3^2} + (1-T)\frac{U_1 U_3}{U_1^2 + U_3^2}\,\frac{\partial U_3/\partial x_2}{\partial U_1/\partial x_2}\right] \tag{3}$$

$$C_{\mu 32} = 0.09\left[(1-T)\frac{U_1 U_3}{U_1^2 + U_3^2}\,\frac{\partial U_1/\partial x_2}{\partial U_3/\partial x_2} + \frac{U_3^2 + TU_1^2}{U_3^2 + U_1^2}\right] \tag{4}$$

The parameter T is given by

$$T = \frac{\tan(\gamma_t - \gamma_v)}{\tan(\gamma_g - \gamma_v)} \tag{5}$$

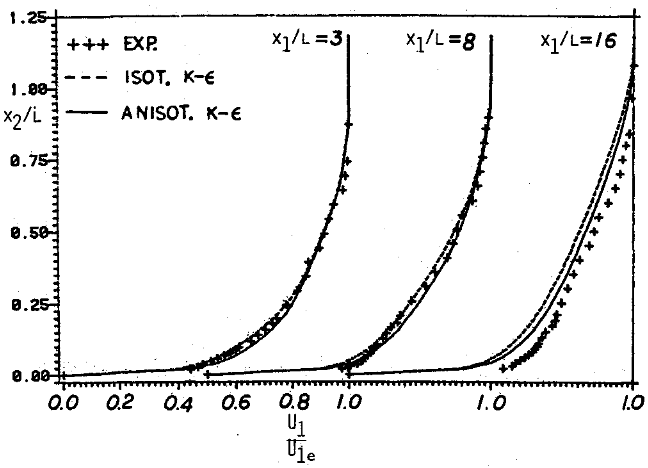

Fig. 1 Comparison between predictions and experiment for the axial velocity profiles.

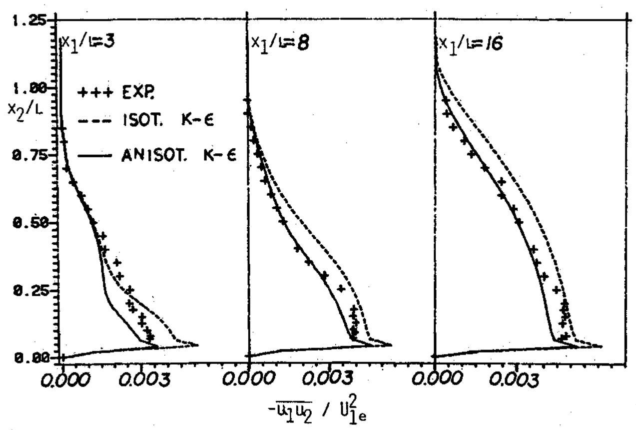

Fig. 2 Comparison between predictions and experiment for stress $\overline{u_1 u_2}$.

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γ_v, γ_t, and γ_g are angles made by the resultant velocity, shear stress, and mean rate of strain vector with the principal direction x, respectively. Note that $T=1$ implies isotropic eddy viscosity.

In the present study, T is assigned an initial value at the initial plane and is then calculated from the local values of γ_v, γ_t, and γ_g as the flow evolves downstream. At each streamwise marching step, after the solution of the mean and turbulent quantities is completed, T is calculated at each point in the boundary layer. This value of T is used in the solution of the next streamwise step. The calculations showed that the variation of T across the boundary layer was not substantial. Therefore, at each streamwise step, an averaged value of T is used for all points in the boundary layer.

The anisotropic k-ϵ model was tested for the flow over Lohmann's[3] rotating cylinder. The mean flow equations and the k and ϵ equations were solved with a space-marching algorithm described in Ref. 4.

T was assigned a value different from 1 at the location where the surfaced started rotating. Beyond that location, it was calculated by the algorithm. The evolution of T for four different initial values of T (0.9, 0.75, 0.5, 0.25) indicated that after an initial adjustment region, T is found to be identical for all initial T values.

Figures 1 and 2 show the comparison between the experimental data and the predictions from the isotropic and anisotropic k-ϵ model at three locations $x_1/L=3$, 8 and 16. $x_1/L=0$ is the location where the rotation starts and L the characteristic length taken to equal to 0.0254 m. It can be seen that the predictions for U_1 and $\overline{u_1u_2}$ have been improved by the use of the anisotropic eddy viscosity model.

Modifications for Rotation

Galmes and Lakshminarayana[5] developed an algebraic stress model for rotating curved flows. In a Cartesian coordinate system, for equilibrium flows ($P=\rho\epsilon$), their model reduces to

$$\frac{\overline{u_iu_j}}{k}=\frac{2}{3}\delta_{ij}+\frac{R_{ij}[1-(C_2/2)]+[P_{ij}-(2/3)\delta_{ij}P](1-C_2)}{C_1P} \tag{6}$$

where

$$R_{ij}=-2\rho\Omega_p(\epsilon_{ipk}\overline{u_ju_k}+\epsilon_{jpk}\overline{u_iu_k})$$

and ϵ_{ijk} is the alternating tensor. For a two-dimensional channel flow rotating about an axis perpendicular to it, Eq. (6) yields (see Ref. 2).

$$-\overline{u_1u_2}=\left(0.09+\frac{\Omega}{\partial U_1/\partial x_2}\right)\frac{k^2}{\epsilon}\frac{\partial U_1}{\partial x_2} \tag{7}$$

The above expression implies that

$$C_\mu=0.09+\Omega\Big/\frac{\partial U_1}{\partial x_2} \tag{8}$$

The modified k-ϵ model was tested the flow in a rotating duct investigated experimentally by Johnston et al.[6] The flow was two-dimensional at the midspan.

The comparison between the predicted and the measured axial velocity is shown in Fig. 3. Even though the agreement with the data is not very good, it can be seen that the modified k-ϵ model correctly predicts qualitatively the asymmetry about the duct centerline ($x_2/2D=0.5$).

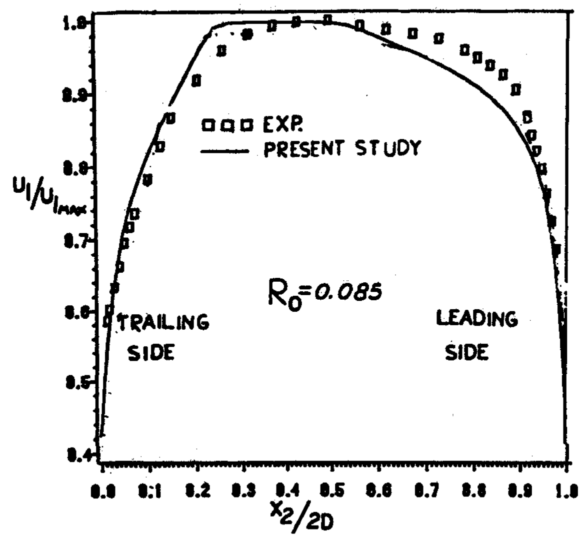

Fig. 3 Comparison between predictions and experiment for the streamwise velocity for $R_0=0.085$.

Conclusions

Modifications to the k-ϵ model to account for the nonalignment of the shear stress vector with the mean strain vector and for the effcts of rotation have been proposed and tested. The modified T model gave encouraging results for the flow over a rotating cylinder, but it needs further testing in more complex three-dimensional boundary layers. The study showed that the rotation effects can be accounted through the modified coefficient C_μ. The present study has illustrated how the k-ϵ model can be extended, with the help of the algebraic stress equations, to account for complex phenomena.

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